



Coherence Function: Theory and VEE applications

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Description: The design of electronic systems, to process analog signals, is always supported by the hypothesis that the system can be considered isolated, linear and time invariant.

In this situation, it is correct to assume that the stimulus used is only the one that is applied to the input and, particularly, the response is due only to the applied input.

The transfer function, obtained as output-input ratio and expressed in modulo and phase over a band of frequency, supplies all the correct information about the system under test.

Equipment:

Agilent 33120 Function Generator
Agilent 54600 Oscilloscope
Agilent 82335B – GP-IB card for P.C.
Agilent-VEE software

1) Coherence function

The design of electronic systems, to process analog signals, is always supported by the hypothesis that the system can be considered isolated, linear and time invariant.

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The transfer function, obtained as output-input ratio and expressed in modulo and phase over a band of frequency, supplies all the correct information about the system under test.

In real situations, the output of electronic systems can present some additional spectral components that the input signal does not present. These components generate an error in the transfer function calculation.

Some examples of these errors can be internal noise in the system, introduction in the system of external noise or signals, and system non-linearity.

The model of this behavior is shown in Fig.1.

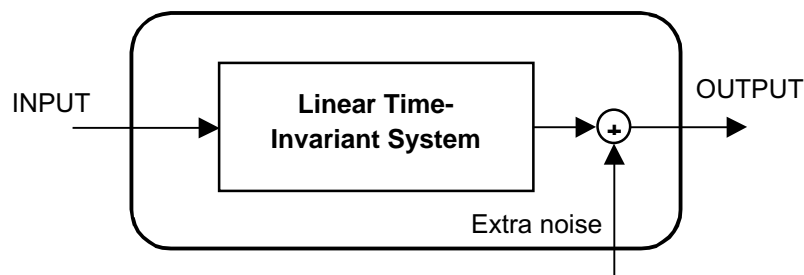


Fig. 1: Model of real system



A good method to investigate these problems consists in the use of the cross spectrum function with an adequate level of linear averaging.

Very efficient and accurate is the coherence function: this function supplies a good estimation of the output energy entity exclusively due to the input signal.

The coherence function is a dimensionless function of frequency having only the real part and values in the range 0 to +1.

At each frequency, the maximum value is present at frequencies really applied to the input while the minimum value is present at frequencies overlapped to the output and, therefore, not correlated to the input signal. In other words, coherence function is the fraction of the output power only due to the input signal:

$$\gamma^2(f) = \frac{\text{Output power only due to the input}}{\text{Total out power}}$$

Let's assume that $f(t)$ and $g(t)$ are the input and output signals of the system shown in Fig. 1 and, $F(f)$ and $G(f)$ the related Fourier transforms. The coherence function $\gamma^2(f)$ is mathematically defined as:

$$\begin{aligned} \gamma^2(f) &= \frac{|S_{FG}(f)|^2}{S_{FF}(f) \cdot S_{GG}(f)} = \\ &= \frac{|F^*(f) \cdot G(f)|^2}{[F^*(f) \cdot F(f)] \cdot [G^*(f) \cdot G(f)]} = \\ &= \frac{[\text{Re}(F(f)) - J\text{Im}(F(f))] \cdot [\text{Re}(G(f)) + J\text{Im}(G(f))]}{[\text{Re}^2(F(f)) + \text{Im}^2(F(f))] \cdot [\text{Re}^2(G(f)) + \text{Im}^2(G(f))]} = \\ &= \frac{[\text{Re}(F(f)) \cdot \text{Re}(G(f)) + \text{Im}(F(f)) \cdot \text{Im}(G(f))] + J[\text{Re}(F(f)) \cdot \text{Im}(G(f)) - \text{Im}(F(f)) \cdot \text{Re}(G(f))]}{[\text{Re}^2(F(f)) + \text{Im}^2(F(f))] \cdot [\text{Re}^2(G(f)) + \text{Im}^2(G(f))]} \equiv 1 \end{aligned}$$

This result can be separated in two real and imaginary parts, and so written:

$$\text{Re}(\gamma^2) = \frac{\text{Re}^2[S_{FG}(f)]}{S_{FF}(f) \cdot S_{GG}(f)} \Rightarrow$$

$$\text{Im}(\gamma^2) = \frac{\text{Im}^2[S_{FG}(f)]}{S_{FF}(f) \cdot S_{GG}(f)} \Rightarrow$$

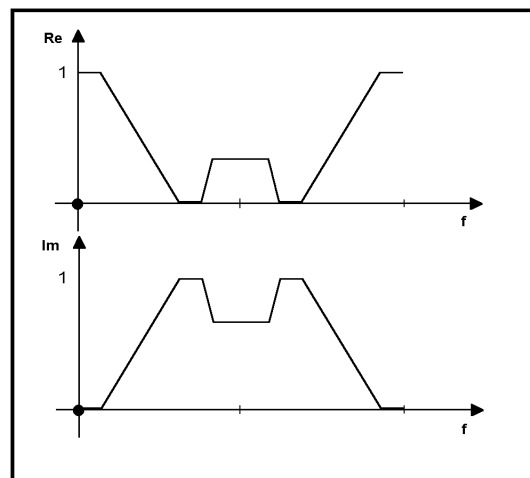


Fig. 2: Real and imaginary part of coherence function



From Fig. 2, it is also possible to confirm that the addition $[\text{Re}(\gamma^2) + \text{Im}(\gamma^2)]$ is identically equal to one for each frequency value.

In real applications, the equation used to calculate the coherence function is:

$$\text{Re}(\gamma^2) = \frac{\text{Re}^2 [S_{FG}(f)]}{S_{FF}(f) \cdot S_{GG}(f)} = \frac{[\text{Re}(F(f)) \cdot \text{Re}(G(f)) + \text{Im}(F(f)) \cdot \text{Im}(G(f))]^2}{[\text{Re}^2(F(f)) + \text{Im}^2(F(f))] [\text{Re}^2(G(f)) + \text{Im}^2(G(f))]}$$

Values of the coherence function less than one are possible if some of the following situations occur:

- A) Contaminated measurements by non correlated noise
- B) Contaminated measurements by additional external signal source
- C) System non linearity
- D) Additional inputs present in the system
- E) Error leakage not reduced with windowing.

The FFT spectrum analyzer provides the computation and display of the coherence function in conjunction with the output power spectrum to allow an efficient comparison between the coherence function variations and the related output spectra components.

To obtain a good accuracy of coherence function measurements, it is very important to use a correct level of frequency domain linear averaging. Higher is the number k of records used and more accurate is the result. In the average, the ratio of the periodic components to the random components is proportional to \sqrt{k} . This means a signal to noise ratio improvement of $10 \cdot \log(k)$ dB.

In modern spectrum analyzers the average level can be set in the 8 to 512 range.

The use of very high values of k implies an increment of computation time and of the necessary memory, therefore it is necessary to know in advance the accuracy level obtainable with a specific value of k , to optimize the measurement process.

To quantify the accuracy, the "90% Confidence Limits", as function of k , is used.

This means that, in 90% of the measures, the most probable value lies in the limits shown in the following table.

Measured value of coherence function	Average $k=16$		Average $k=32$		Average $k=64$		Average $k=128$		Average $k=256$	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
0.40	0.15	0.59	0.23	0.54	0.28	0.50	0.32	0.47	0.34	0.45
0.50	0.25	0.67	0.33	0.63	0.39	0.59	0.42	0.57	0.45	0.55
0.60	0.36	0.74	0.45	0.71	0.50	0.68	0.53	0.66	0.55	0.64
0.70	0.50	0.81	0.57	0.78	0.61	0.76	0.64	0.75	0.66	0.73
0.80	0.65	0.88	0.70	0.86	0.74	0.84	0.76	0.83	0.77	0.82
0.90	0.81	0.94	0.85	0.93	0.87	0.92	0.88	0.92	0.88	0.91

Tab. 1: 90% Confidence Limits on coherence function measurements



As shown in the definition of coherence function, the term $\gamma^2(f) \cdot S_{GG}(f)$ is the output power related to the input signal, and the term $[1 - \gamma^2(f)] S_{GG}(f)$ is the noise component of the output power, therefore, the signal to noise ratio of the system under test is so computable:

$$\frac{S}{N}(f) = \frac{\gamma^2(f)}{1 - \gamma^2(f)}$$

Note that, the signal to noise ratio is a function of frequency and not the overall.

2) VEE flow diagram

A powerful tool to simulate a coherence function measure is VEE software.

Fig. 3 shows a VEE flow diagram to calculate a coherence function of a theoretical L.T.I system.

The two blocks S1 and S2 are two signal generators, (S1 + S2) block represents the input of the system.

The N1 block is a random noise generator and N2 block is a deterministic signal noise.

The block [(S1+S2) + (N1+N2)], represent the output of the system.

There are two identical set blocks 0, JCT, A+B and A/B: the first executes the frequency domain linear average of the coherence function and the second executes the frequency domain linear average of the power spectrum function, both to be displayed.

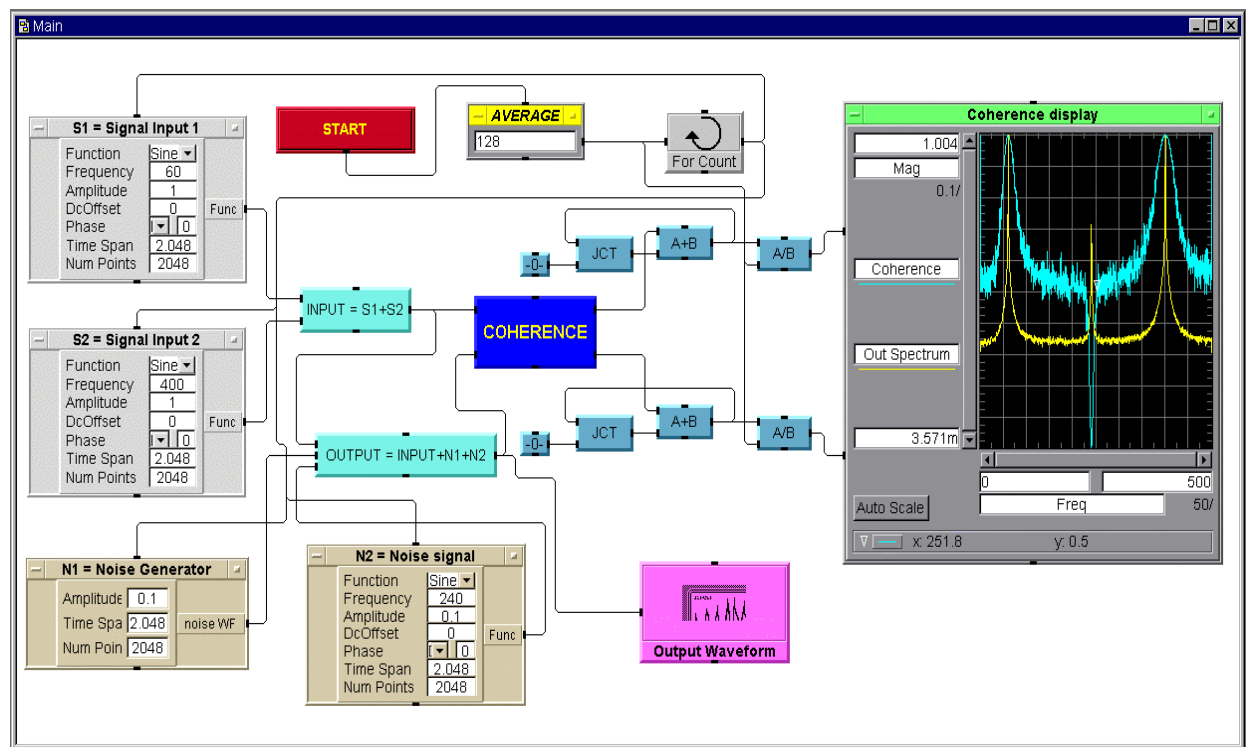


Fig. 3: VEE flow diagram to calculate coherence function



The COHERENCE block is a User Object dedicated to coherence function calculation.

Fig. 4 shows a VEE flow diagram of this User Object.

This block executes the equation used to calculate the coherence function and the dB power spectrum, normalized to the 0 ÷1 interval which is the same scale used by coherence function.

Experiment set-up of Fig. 3:

- INPUT: Sinewave(1 volt, 60 Hz) + Sinewave(1 volt, 400 Hz)
- OUTPUT: INPUT + RandomNoise(0.1 volt) + Sinewave(0.1 volt, 240 Hz)
- Sample Rate: 1000 Hz
- Record Length: 2048 samples
- Average Level: 128

The coherence display shows high values (about 1) of coherence function to frequency values of 60 and 400 Hz, and low values of coherence function (about 0) to frequency value of 240 Hz. The effect of the random noise is limited by the high level (128) of linear average.

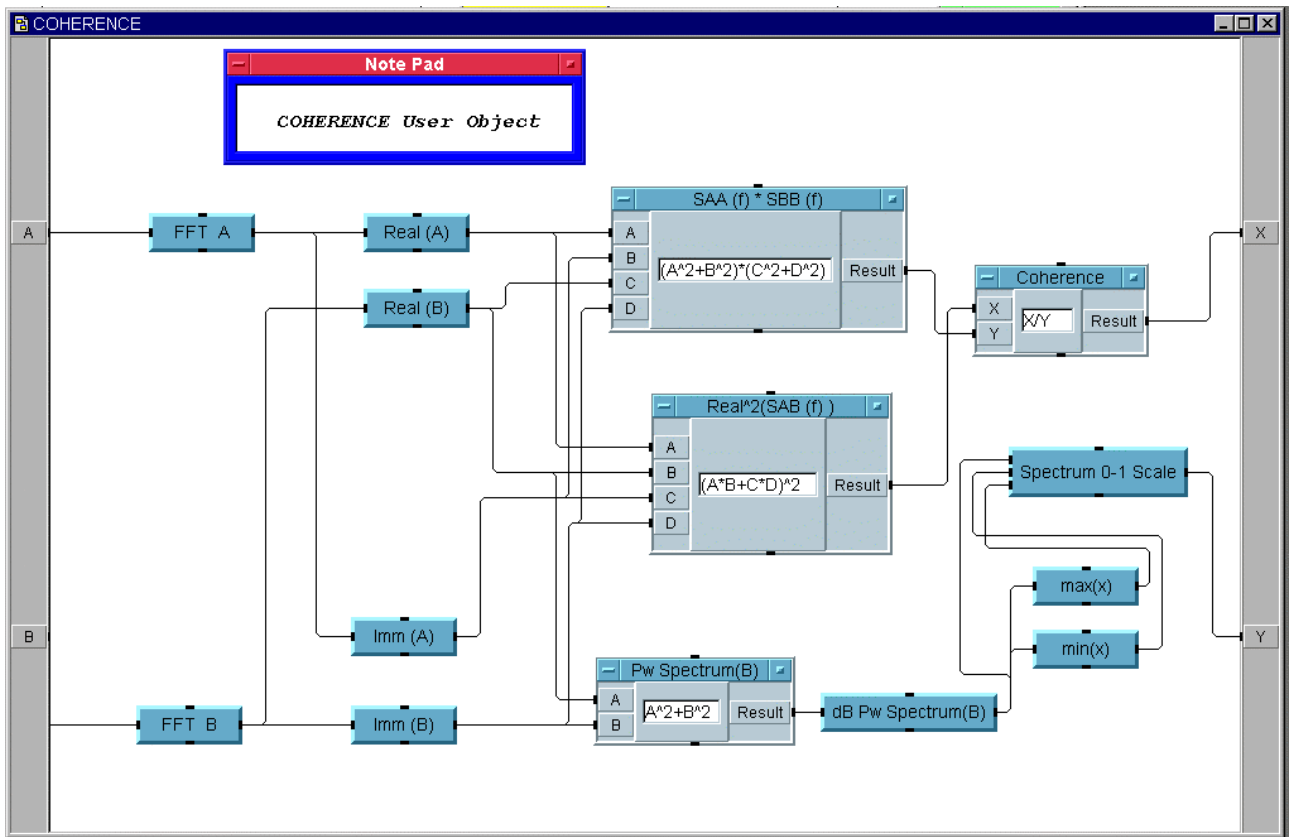


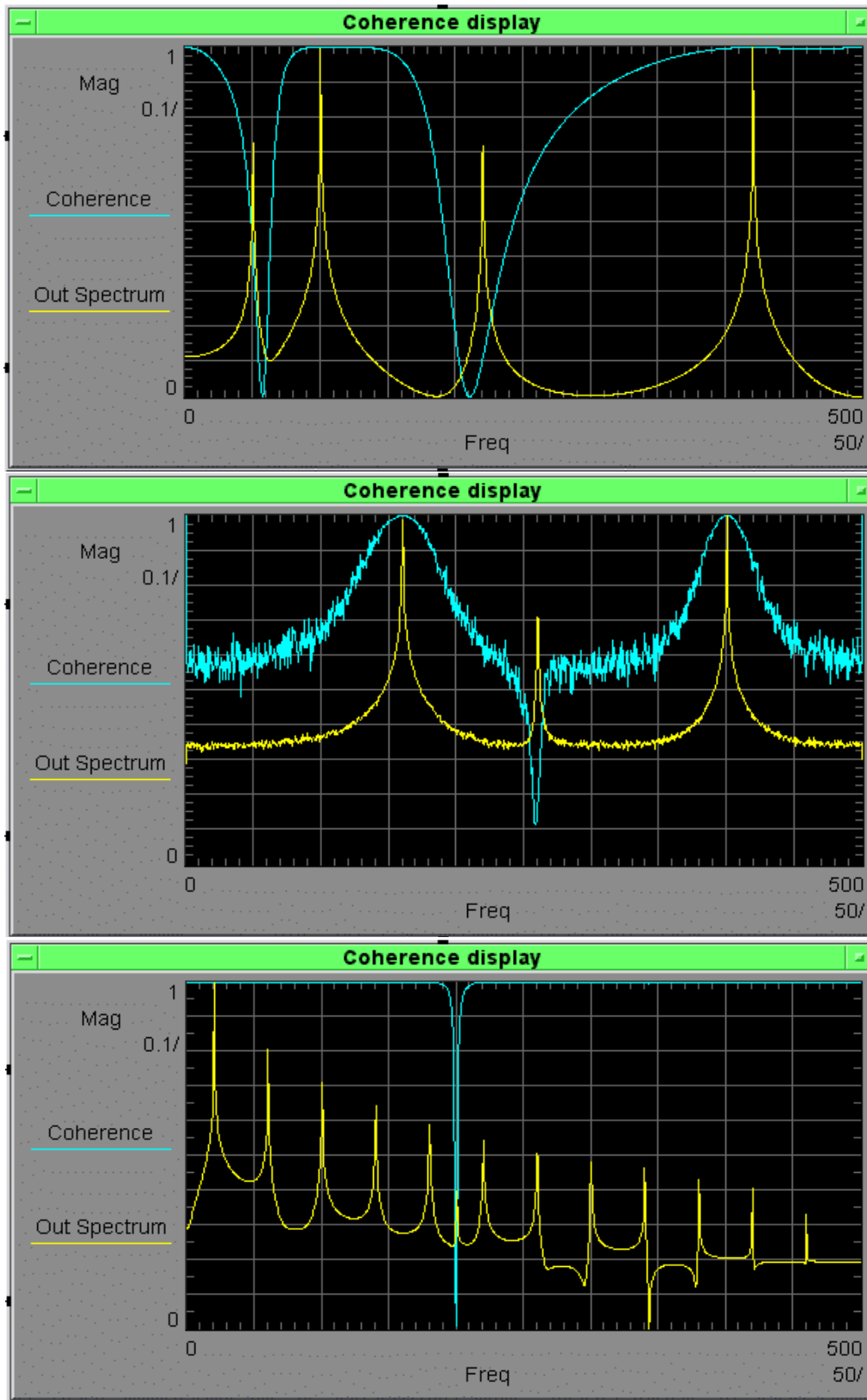
Fig. 4: Coherence User Object



Some examples of coherence function calculation are shown in the following Fig. 5 and Fig. 6 displayed with the relative output power spectrum function.

The experiments set-up are so defined:

A	IN = Sinewave(1 volt, 100 Hz) + Sinewave(1 volt, 420 Hz) OUT = IN + Sinewave(0.2 volt, 50 Hz) + Sinewave(0.2 volt, 220 Hz) Record Length = 2048 samples Average Level = 4
B	IN = Sinewave(1 volt, 160 Hz) + Sinewave(1 volt, 400 Hz) OUT = IN + RandomNoise(0.1 volt) + Sinewave(0.1 volt, 260 Hz) Record Length = 2048 samples Average Level = 128
C	IN = TriangularWave(1 volt, 20 Hz) OUT = IN + Sinewave(0.001 volt, 200 Hz) Record Length = 2048 samples Average Level = 8
D	IN = Sinewave(1 volt, 20 Hz) OUT = IN ² (non linear system) Record Length = 2048 samples Average Level = 8
E	IN = Sinewave(1 volt, 200 Hz) OUT = IN + RandomNoise(0.1 volt) + Sinewave(0.3 volt, 205 Hz) Record Length = 2048 samples Average Level = 256
F	IN = Sinewave(1 volt, 85 Hz) OUT = IN + RandomNoise(0.5 volt) + Sinewave(0.3 volt, 60 Hz) + Sinewave(0.3 volt, 120 Hz) Record Length = 4096 samples Average Level = 512
Common sample rate = 1000 Hz	

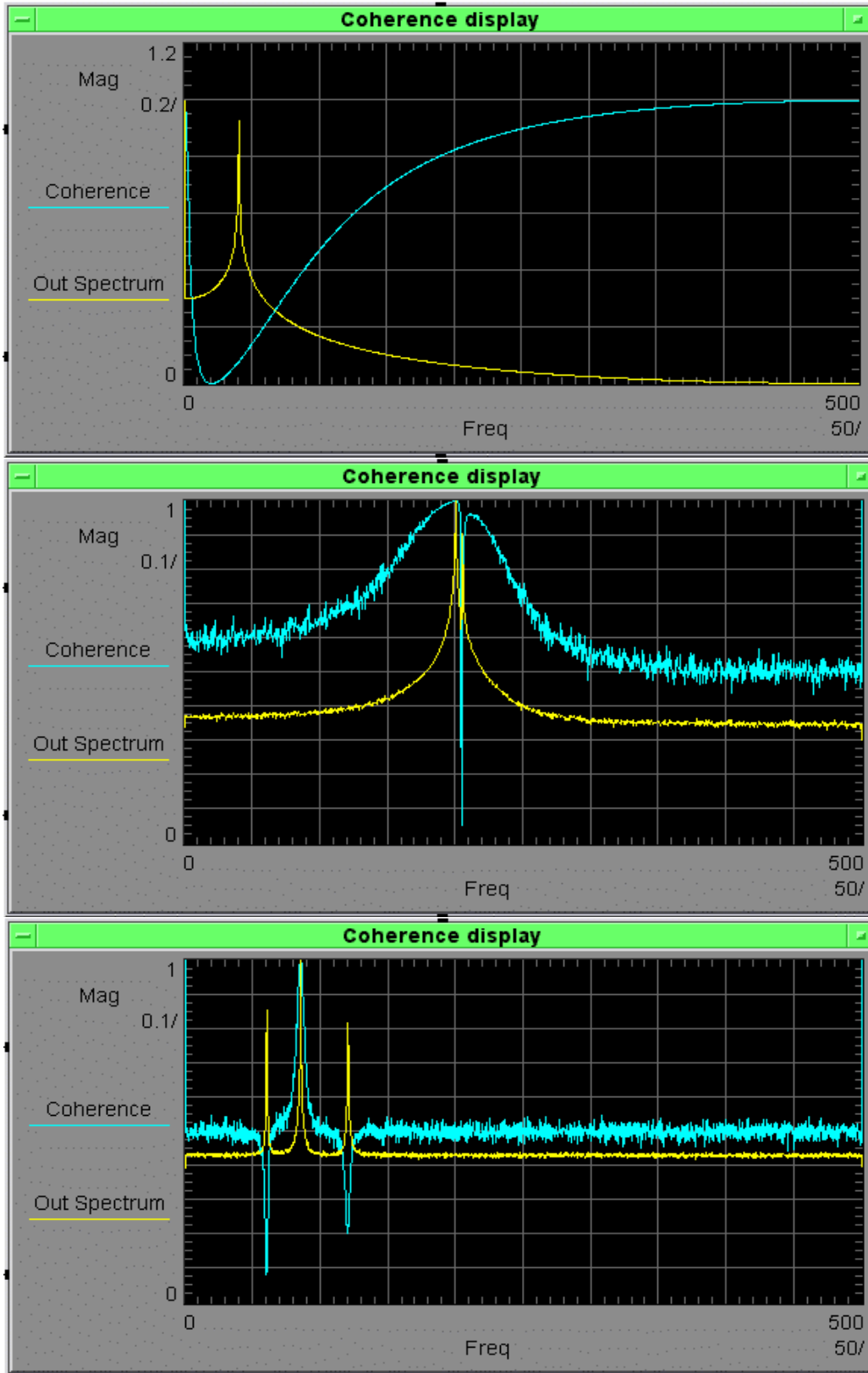


A

B

C

Fig. 5: Examples A, B and C of Coherence Function calculation



D

E

F

Fig. 6: Examples D, E and F of coherence function calculation



3) System measurement implementation

It is possible to execute the measurement of coherence function by using an oscilloscope with an IEEE488 interface connected to a PC with an IEEE488 interface card such as the Agilent 82335.

The block diagram of the measurement bench is shown in Fig. 7.

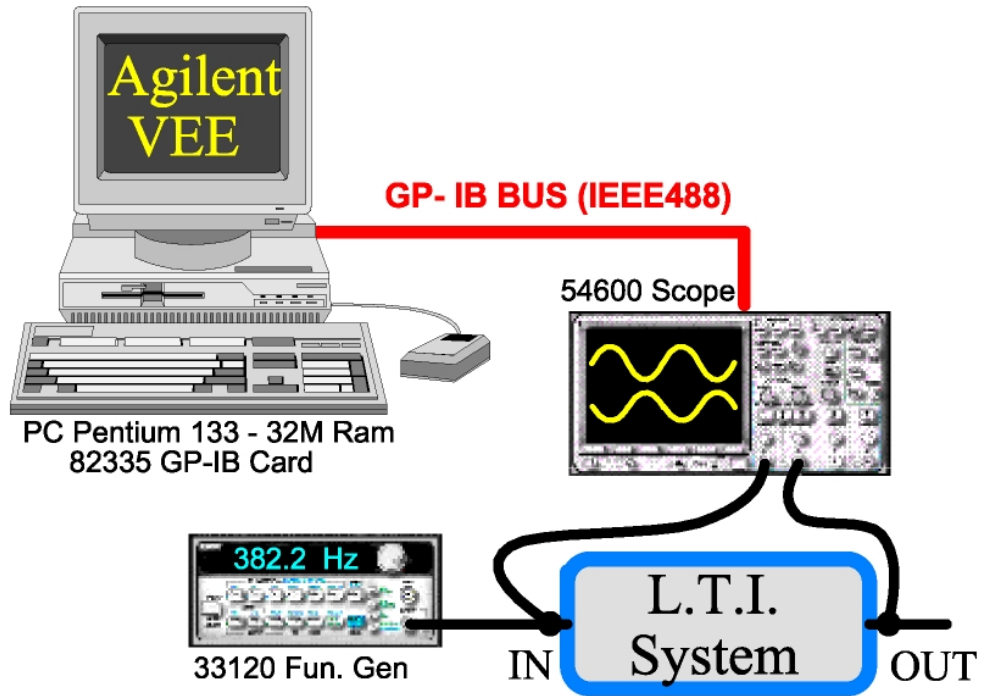


Fig. 7: Block diagram of the measurement bench

The function generator is used to drive the system under test with the correct signal input.

The PC acquires, from the oscilloscope, the sampled records to calculate the coherence function by means of the VEE program shown in Fig. 3.